Polynomial Expressions and Functions

These notes are intended as a summary of section 1.1 (p. 2 – 6) in your workbook. You should also read the section for more complete explanations and additional examples.

Division of a Polynomial by a Binomial

Use long division to determine each quotient. Verify the answer.

$2748 \div 13$ $\qquad\qquad$ $4212 \div 27$

Long division can be used to divide a polynomial by a binomial. Since division by zero is not possible, assume that the divisor is never 0.

For example, to divide $3x^2 - 4x + 5$ by $x - 2$:

$$\begin{array}{r} \textit{Quotient} \\ \textit{Divisor}\overline{\smash{)}\textit{Polynomial}} \end{array}$$

$$x - 2\overline{\smash{)}\,3x^2 - 4x + 5}$$

Let’s try a few more examples.

Divide $2x^3 - x^2 - 2x + 1$ by $x + 1$:

$$x+1\overline{\smash{\big)}\ 2x^3 - x^2 - 2x + 1}$$

Divide $x^2 + 2x + 5$ by $x - 1$:

$$x-1\overline{\smash{\big)}\ x^2 + 2x + 5}$$

Note: Before dividing, we must write the polynomial and the binomial in descending order.

Example 1 (sidebar p. 3)
Divide: $2x^3 + 5 - 2x + 3x^2$ by $x - 1$

Division Statements

A division statement relates the original polynomial and the divisor to the quotient and the remainder. It has the form:

$$Polynomial = Divisor \times Quotient + Remainder$$

This is often written in function notation as:

$$P(x) = (x - a) \cdot Q(x) + R$$

For example, the division statement for Example 1 would be

$$2x^3 + 5 - 2x + 3x^2 = (x - 1)(2x^2 + 5x + 3) + 8$$

Let's go back and write division statements for all of our examples that we've done so far.

Note: Sometimes a polynomial will be missing terms that contain some power of the variable. For example, $-4x^4 + 2x^2 - x - 3$ has no x^3 term. In such cases, we insert a term with a coefficient of zero (e.g. $0x^3$) as a placeholder.

Example 2 (sidebar p. 5)
Divide: $3x^4 - x^3 + 3x - 20$ by $x + 2$. Write the division statement.

Homework: #3, 5 – 7 in the exercises (p. 7 – 12). Answers on p. 13.